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Fight the power: Lanchester's laws of combat in human evolution

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ABSTRACT

Lanchester's "Laws of Combat" are mathematical principles that have long been used to model military conflict. More recently, they have been applied to conflict among animals, including ants, birds, lions, and chimpanzees. Lanchester's *Linear Law* states that, where combat between two groups is a series of one-on-one duels, fighting strength is proportional to group size, as one would expect. However, Lanchester's *Square Law* states that, where combat is all-against-all, fighting strength is proportional to the *square* of group size. If conflict has been important in our evolutionary history, we might expect humans to have evolved assessment mechanisms that take Lanchester's Laws of Combat into account. Those that did would have reaped great dividends; those that did not might have made a quick exit from the gene pool. We hypothesize that: (1) the dominant and most lethal form of combat in human evolutionary history (as well as among chimpanzees and some social carnivores) has been asymmetric raids in which multiple individuals gang up on a few opponents, approximating Square Law combat; and (2) this would have favored the natural selection of an evolved "Square Law heuristic" that correlated fighting strength not with raw group size but with group size *squared*. We discuss the implications for primate evolution, human evolution, coalitionary psychology, and contemporary war.

1. Introduction

> "Words are inadequate to describe the emotion aroused by the prolonged movement in unison that drilling involved. A sense of pervasive well-being is what I recall; more specifically, a strange sense of personal enlargement; a sort of swelling out, becoming bigger than life, thanks to participation in a collective ritual."
>
> [William McNeill (1995, p. 2)]

> "We've got them!"
>
> George Armstrong Custer, at the Battle of the Little Bighorn.
>
> [Stephen Ambrose (1975, p. 438)]

On 2 August 1867, Crazy Horse led a force of one thousand Sioux warriors in an attack on a US Army outpost near Fort Phil Kearny in northern Wyoming. Captain J. N. Powell gathered 26 soldiers and a handful of armed civilians in a corral of wagons, and they prepared to defend themselves. The Sioux initially circled Powell's position on horseback, firing arrows, intending to exhaust the cavalrymen's ammunition, but to no avail. Powell had stockpiled several thousand rounds, and the soldiers kept up a constant hail of fire. Eventually, Crazy Horse pulled his warriors back into a ravine, where they were partially protected from the gunfire. From here, the Indians attempted to attack on foot. The ravine was narrow which, as Stephen Ambrose describes, meant that "the men in front masked the mass of warriors in the rear, making it impossible for them to fire … Powell only had to deal with a handful of Indians, Crazy Horse and his fellow shirt-wearers [Sioux leaders] at the apex of the charge" (Ambrose, 1975, pp. 294–295). At this point, the outcome of the battle remained far from certain to those present. As one soldier recounted, "It chilled my blood … Hundreds and hundreds of Indians swarming up a ravine about ninety yards [away]… Our fire was accurate, coolly delivered and given with most telling effect, but nevertheless it looked for a minute as though our last moment on earth had come" (Ambrose, 1975, p. 295). Against their volleys of arrows and some astonishingly brave charges, the withering fire from the cavalry's new breech-loading rifles wore the Indians down and, after several hours' fighting, they withdrew to the mountains.

Against the backdrop of the earlier Fetterman massacre of 1866, when Crazy Horse and two thousand Sioux had surrounded Captain William Fetterman's force of 81 cavalrymen and annihilated them to a man, Powell's victory against the odds seemed nothing less than a miracle. But the reason Powell lived to see another day may well have been down to some fundamental mathematical principles of battle. Crazy Horse's congested attack up the ravine meant he was not able to bring his superior numbers and their deadly arrows to bear—even on a tiny enemy force. Meanwhile, Powell's concentrated fire on the lead ranks of Indians meant that, despite Powell's force being outnumbered 25 to 1, any Indian that squeezed onto the frontline fell into the sights of several American soldiers at once. Despite Crazy Horse's numerical

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supremacy and the advantage of surprise, the deck was stacked against him.

The "Wagon Box Fight" of 1867 reflects the mathematical patterns of Lanchester's Laws of Combat (Lanchester, 1916). These "laws" are mathematical equations that model the dynamics of conflict and its outcomes, and were originally developed with modern human warfare in mind. Although they have long been used in military operational research (for reviews, see MacKay, 2006; Wrigge, Fransen, & Wigg, 1995), they have only recently been applied to explain variation in the patterns of conflict in animals such as ants, birds, lions, and chimpanzees (Franks & Partridge, 1993; Mosser & Packer, 2009; Plowes & Adams, 2005; Shelley, Tanaka, Ratnathicam, & Blumstein, 2004; Whitehouse & Jaffe, 1996), including manipulation experiments showing variation in fighting behavior as parameters were changed (McGlynn, 2000; Wilson, Britton, & Franks, 2002).

Much of the literature on Lanchester's Laws looks at models and data with regard to combat *outcomes*. In this paper we make a rather different kind of argument. First, we argue that Lanchester's Square Law, under which imbalances in numbers are disproportionately advantageous to the larger side, is especially applicable to pre-military human conflict, and is likely to have influenced its dynamics for several million years. This provides substantive support to theories about the importance of human groups and coalitions in early warfare (Alexander, 1987; Bingham, 2000; Pitman, 2011; Wrangham, 1999a).[1]

Second, the question then naturally arises: Have we evolved corresponding assessment strategies that influence when (and how) we choose to fight? Violent conflict is argued to have played a major role in our ancestral past (Buss & Shackelford, 1997; Ferguson, 2012; Gat, 2006; Guilaine & Zammit, 2004; Keeley, 1996; LeBlanc & Register, 2003; Potts & Hayden, 2008; Wrangham & Peterson, 1996, though for an earlier, contrasting view see Knauft 1991). Empirical studies suggest that warfare accounted for around 15% of male deaths among archeological and ethnographic data (and much more in some societies Bowles, 2006; Keeley, 1996; Otterbein, 1989), implying strong selection pressure on adaptations for fighting—and winning. We therefore hypothesize that natural selection should have favored assessment mechanisms that take the Square Law into account, leading to an evolved "Square Law heuristic" in the context of coalitionary conflict. Thus the Square Law becomes more than a *post hoc* model of conflict outcomes: rather it may be an evolved heuristic that influences decisions about whether or not to fight in the first place, continuing to affect decisions about conflict today. If so, this carries major implications for understanding human conflict in our past, present, and future.

2. Lanchester's Laws of Combat

Although there are variations in how the models are set up, and in real life there are many complicating factors (Adams & Mesterton-Gibbons, 2003; Johnson & MacKay, 2011; MacKay, 2011), the underlying logic of Lanchester's Laws capture the essence of conflict processes irrespective of species or setting—"elementary principles", as Lanchester called them, "which underlie the whole science and practice of warfare in all its branches" (Lanchester, 1916, p. 39). The key insight is the distinction between the Linear Law and the Square Law.

2.1. Lanchester's Linear Law

Consider two opposing sides with m individuals in the blue force and n individuals in the red force (we follow the notation of Adams & Mesterton-Gibbons, 2003), in hand-to-hand combat along a battle line. If α denotes the fighting ability of individuals, then the attrition rate for the blue force is

$$\mathrm{d}m/\mathrm{d}t = -\alpha_n l, \tag{1}$$

while for the red force

$$\mathrm{d}n/\mathrm{d}t = -\alpha_m l, \tag{2}$$

where l is the length of the battle-line, representing the number of individuals on each side actually engaged in the fighting. The crucial feature is that this is the same for each side, for example $l = \mathrm{Min}(m,n)$ (*i.e.* the number in the smaller of the two forces, though it may be constrained by some other factor such as the available space in which to fight). Nor do we need to know the precise form of l in order to predict the battle's outcome. If we divide Eq. (1) by Eq. (2), the explicit time-dependence disappears, as does the dependence on l, and we have

$$\mathrm{d}m/\mathrm{d}n = \alpha_n/\alpha_m, \tag{3}$$

so that the casualty ratio $\mathrm{d}m/\mathrm{d}n$ is constant.[2] Rearranging and integrating (which corresponds to summing over all the small changes that combine to determine the outcome) we obtain

$$\alpha_m(m_0 - m) = \alpha_n(n_0 - n), \tag{4}$$

where m_0 and n_0 are the initial numbers of blue and red soldiers. Thus m wins if

$$\alpha_m m_0 > \alpha_n n_0. \tag{5}$$

Following Lanchester, we call this combination of numbers and prowess (in this case, simply their product) the "fighting strength" of a given group, so that the force with the greater fighting strength wins the battle. In this, Lanchester's Linear Law, fighting strength is *proportional* to fighting ability (α) and *proportional* to group size (m).

2.2. Lanchester's Square Law

Here's where it gets interesting. Consider two opposing sides as before. This time, attrition rates for the blue force are

$$\mathrm{d}m/\mathrm{d}t = -\alpha_n n \tag{6}$$

and for the red force

$$\mathrm{d}n/\mathrm{d}t = -\alpha_m m. \tag{7}$$

The difference is that, in this fight, combat is not restricted—there is no battle line, no set of duels, no one unable to get into the fight. Rather, each force can engage all its soldiers, and thereby cause enemy losses in proportion to its own numbers. For Lanchester, this was the defining property of war characterized by accurate, aimed projectile fire (such as rifles). But such conditions occur more generally whenever some form of "ganging up" is possible.

Now we again divide one equation by the other, obtaining

$$\mathrm{d}m/\mathrm{d}n = (\alpha_n/\alpha_m)(n/m). \tag{8}$$

In contrast to Eq. (3), the casualty ratio is not constant, but rather is proportional to the force ratio (to be clear, the "force ratio" being n/m). This has stark effects when we rewrite Eq. (8) as

$$\alpha_m m \,\mathrm{d}m = \alpha_n n \,\mathrm{d}n, \tag{9}$$

[1] For recent collections on the evolution of human violence more generally see Shackelford and Hansen (2014) and Fry (2013).

[2] Of course, in real hand-to-hand pitched battles casualty numbers often are hugely asymmetric, usually because most casualties occur in the rout of fleeing troops rather than in the battle line, a point recognized by Lanchester. They also depend on the skill of the soldiers, as detailed below.

relating the instantaneous losses of the two forces, and integrate, for then we find that

$$\alpha_m \left({m_0}^2 - m^2 \right) = \alpha_n \left({n_0}^2 - n^2 \right). \quad (10)$$

Thus m wins if

$$\alpha_m {m_0}^2 > \alpha_n {n_0}^2. \quad (11)$$

This is Lanchester's Square Law. Fighting strength is still proportional to fighting ability (α), as in the Linear Law, but is now proportional to the *square* of group size (m^2). In situations where the more numerous side can make its superior numbers tell, bringing everyone into the battle and ganging up on opponents, numerical advantage becomes disproportionately important.

2.3. Implications of the Square Law

Let us illustrate this remarkable result. In a single fight between equal numbers – say 100 blues against 100 reds – the outcome is the same under either law: the side whose individuals have the better weapons or greater fighting ability (α) wins. It is only when there is an asymmetry in the numbers on each side (i.e., a force ratio differing from 1) that Lanchester's logic bites. The Square Law generates two crucial effects in particular.

First, suppose that blue individuals have a 30% greater fighting ability than reds, but red has 20% greater numbers. Under the Linear Law, blue wins, since $1.2 < 1.3$. But under the Square Law, the winner is now red, since $(1.2)^2 > 1.3$. Small differences in numbers can easily trump larger differences in skill.

Second, Lanchester's Laws discriminate varying conditions in a single, simultaneous fight. But now suppose 100 blues meet 100 reds, all of identical fighting ability, not in one but in two, sequential fights. Under the Linear Law, the outcome is mutual annihilation, whatever the nature of the split. But under the Square Law, things are very different. Suppose the 100 blues first fight 50 reds and then, once these are beaten, the other 50. Then blue dispatches the first 50 reds with the loss of only 13 blues, leaving 87 blues to dispatch the remaining 50 reds. Blue finishes with 71 survivors in, all else equal, an "easy and decisive victory" (Lanchester, 1916, pp. 42–43). Here, ganging up tells.

The reason for the disproportional advantage in a Square Law setting is that the larger group can concentrate multiple attacks on individuals of the weaker side. In contrast, the weaker side's individual efforts are spread thinly against their more numerous opponents.[3] The two effects combine to give the Square Law, which underlies some fundamental military logic (much of which has been around for a long time). For example, consider the principle of "concentration", that one should usually not divide one's forces. In the extreme case of the example above, where blue were able to subdivide the red force into arbitrarily small units, blue would not only win but do so with almost no casualties—red would be "defeated in detail".

A typical dynamic of a battle is that the outnumbered side (intuitively) tries to effect Linear Law conditions, perhaps by fighting shoulder-to-shoulder or back-to-back, or by choosing highly constrained terrain such as a narrow pass or with their backs to a wall. The more numerous side, by contrast, often (intuitively) tries to create Square Law conditions, a homogeneous all-against-all fight where their superior numbers can be brought to bear.

The two classic Lanchester laws can be generalized within a broader class of scaling laws (see e.g. Epstein, 1986). The Linear and Square Laws lie on a spectrum in which hybrids or intermediate laws are perfectly possible and natural. When a force causes losses in proportion to its own numbers, this results in the Square Law, as we have seen. However, when a force also *suffers* losses in proportion to its numbers – for example, in Lanchester's "unaimed fire" model, of incoming random fire causing losses in proportion to the density of targets – this pushes the outcome back towards the Linear Law. Table 1 compares features of Lanchester's Laws of Combat. The key to the laws is the dependence of the casualty ratio on the force ratio (Eqs. (3) and (8)): the Square Law obtains whenever the two are proportional, and that generally means where ganging up is easy.

Table 1
A comparison of key features of Lanchester's Laws of Combat.

	Linear Law	Square Law
Type of combat	One-on-one, duels, fighting numbers constrained	Many-against-few
Ganging up	Not possible	Possible
Attacks	Heavily constrained: random, unaimed, poorly targeted or (for some individuals) impossible	All individuals can make clearly targeted attacks
Casualty ratio	Constant	Proportional to force ratio
Fighting strength proportional to	• Fighting ability • Group size	• Fighting ability • Group size squared
Examples	• Leonides at Thermopylae • Crazy Horse at Fort Kearny	• Nelson at Trafalgar • Crazy Horse at Little Bighorn

A final point, central to our later argument, concerns an individual soldier's own chance of surviving a fight. Suppose his own force has the same individual fighting skills and weapons as its opponents, but outnumbers them by k to 1, and battles them to annihilation. In a Linear Law fight, the soldier's chance of death is $1/k$. But in a Square Law fight it is $1 - \sqrt{(1 - 1/k^2)}$, which for $k \geq 2$ is approximately equal to $1/2k^2$. The contrast is stark: if you outnumber your opponents by 3 to 1, then in a Linear Law battle you still have a 1 in 3 chance of being killed—but in a Square Law battle this falls to 1 in 18. Of course most battles are not fought to annihilation, and often not with the same weapons and skill on each side, but *mutatis mutandis* the point remains: over the course of a Square Law fight, your chances of getting hurt, which you might naively consider to be inversely proportional to the force ratio against you, are in fact much less than that. This remarkable disparity is significant for the argument that a low cost was critical for the evolution of lethal aggression (Manson & Wrangham, 1991), and for solving the collective action problem of warfare, which is considered to be one of the most challenging such problems precisely because of the attendant risk of injury and death (Mathew & Boyd, 2011; Sosis, Kress, & Boster, 2007; Tooby & Cosmides, 1988; Tooby & Cosmides, 2010).

2.4. Historical examples and evidence

Some famous battles in history serve to illustrate the logic of Lanchester's Laws. A classic example of the Linear Law is the battle of Thermopylae in 480 BC (de Souza, 2003). Leonidas led the Spartans to meet a Persian invading army at a narrow mountain pass. The soldiers could only engage across a narrow defile, each rank coming forward as the leading ones fell. This suited the Spartans well, because although massively outnumbered (with tens of thousands of Persian soldiers ranged against their band of a few hundred), they were able to exploit their greater fighting ability (α; see Eq. (5)). In contrast to the largely conscript and mercenary Persian soldiers, the Spartans were famously trained as warriors from birth and veterans of war. The one-on-one duels along a constrained battle line allowed them to keep the mighty Persian force at bay for three days, bringing Thermopylae into legend.[4]

By contrast, Nelson's victory at Trafalgar in 1812 is attributed to his negation of his enemy's Square Law advantage (Lanchester, 1916).

[3] This point was originally made in the context of naval warfare by Baudry (1910).

[4] They only lost when a Greek traitor led a Persian force around a secret mountain path to attack them from behind. Now they were fighting on more than one front, some Greek contingents withdrew, and many succumbed from a deluge of Persian arrows.

Outnumbered in ships and crew, Nelson used two formations of ships to charge the centre of the enemy line, dividing it. He was then able to gang up, two-on-one, on one portion of the enemy fleet (as had previously brought him success in the Battle of the Nile), while the other was held at bay downwind.

Nelson's victory, like Leonides', became legend. Widely studied in military academies, the logic of such successes has become embodied in modern military "principles of war". Thus what may have been, for earlier commanders, subconscious or intuitive exploitation of the workings of Lanchester's Laws of Combat has now become standard doctrine (MacKay & Price, 2011). Lanchester himself noted that "Nelson, if not actually acquainted with the *N-square* law, must have had some equivalent basis on which to figure his tactical values" (Lanchester, 1916, p. 66). Certainly WW I naval leaders rapidly became acquainted with it: Admiral John Jellicoe, commander of the main British fleet comprising well over a hundred ships, wrote to Lanchester that "your N-square law has become famous in the Grand Fleet."[5]

However, evidence for the Square Law at the level of battle outcomes is scant. Early support (Engel, 1954) was superseded by a more confused later picture, in which land, air and all-arms battle outcomes more typically approximate Linear Law results (among numerous papers see, for example, Fricker, 1998; Johnson and MacKay, 2011; Lucas and Turkes, 2004; a full bibliography is given by Wrigge et al. (1995)). The underlying point is that Square Law conditions are not the norm, especially above what Turney-High called the "military horizon" (Turney-High, 1949), where the disadvantaged force can usually work to mitigate the effect. For example, a study of troops under training conditions demonstrates that, with automatic weapons, the combination of fire suppression and poor fire control can actually cause a force to receive hits in proportion to its own rather than its opponents' numbers (Johnson, 1990). Rather, a Square Law advantage is special, often fleeting, and depends crucially both on unequal numbers and on the more numerous force's ability to make its superior numbers tell—that is, to use its superior force ratio to create a proportionately superior ratio of damage inflicted.

3. Which law applied in human evolution?

The dramatic difference between the Linear and Square Laws suggests that humans may have evolved rather different adaptations to conflict, depending on what type of conflict was dominant in human evolutionary history. We discriminate two distinct possibilities in particular:

Hypothesis 1 (H1). If lethal conflict in human evolutionary history was primarily either duel-like (only one individual tended to fight one other at a time, so there was no opportunity for multiple attacks) or in the form of formal pitched battles between comparable numbers (with either hand-to-hand fighting or poorly aimed projectiles), then we would expect assessment strategies that equate fighting strength with group size.

Hypothesis 2 (H2). If lethal conflict in human evolutionary history was primarily in the form of fights between unequal numbers, in which ganging up and multiple, concentrated attacks on enemies were typical, then we would expect assessment strategies that equate fighting strength with group size *squared*.

Of course neither of Lanchester's Laws is going to obtain precisely in the real world. However, as Lanchester succinctly put it: "Superior morale or better tactics or a hundred and one other extraneous causes may intervene in practice to modify the issue, but this does not invalidate the mathematical statement" (Lanchester, 1916, p. 50). To paraphrase Epstein (2007), there is always *some* underlying model. Over evolutionary time, natural selection is likely to have honed adaptive mechanisms that track the underlying mathematics of conflict, however obfuscated they may be in the fog of war.

If lethal conflict in human evolutionary history was a mix of some duel-like and some many-against-few fights, in a hybrid of Hypothesis 1 (H1) and Hypothesis 2 (H2), then we might expect a single, hybrid assessment strategy that scales fighting strength with group size to some power intermediate between 1 and 2 (and thus still imputes some disproportionate advantage to numbers). Alternatively, there might be a flexible assessment mechanism that is able to *switch on* either Square Law assessment *or* Linear Law assessment depending on the situation.

Note that we say "primarily" and "lethal conflict" in the hypotheses above because what matters is not their frequency or rarity, but their impact on costs and benefits to Darwinian fitness. We would only expect to see adaptations to types of conflict that have fitness consequences, and thus exert selection pressure on assessment mechanisms. Thus if Linear Law battles were common, but rarely caused serious injuries or fatalities, whereas Square Law battles were rare, but caused significant injuries or deaths when they did occur, then we might expect adaptations principally to the Square Law. But what support do we have for either hypothesis?

3.1. The imbalance of power hypothesis

Richard Wrangham's "imbalance-of-power hypothesis" (Manson & Wrangham, 1991; Wrangham, 1999a) suggests that lethal combat throughout human evolutionary history, as well as among chimpanzees and some social carnivores, was almost always limited to situations in which an attacking side has an overwhelming numerical advantage and made concentrated attacks on their victims. By ganging together, groups of individuals are able to ambush, pursue, and kill rivals at little cost to themselves (see also Keeley, 1996; Wadley, 2003; Wrangham & Wilson, 2004). In various forms of conflict, this tendency has continued into modern times (Collins, 2008). If so, human evolution has been subject to several million years of Lanchester's Square Law.

The imbalance-of-power logic can be observed, for example, among wolves. Studies of undisturbed populations in Alaska have found that as much as 39%–65% of adult deaths were due to inter-group killing (Mech, Adams, Meier, Burch, & Dale, 1998). The reintroduction of wolves into Yellowstone National Park in the 1990s offered an opportunity to observe inter-group competition more intensively, and asymmetric interactions appear to be the norm (Halfpenny, 2003). A detailed population study extending from 1998 to 2010 found that intra-specific killing was the most common cause of death, accounting for 58 (37%) of the 155 wolves found dead (Cubaynes et al., 2014). While the circumstances of each case varies, typically one side attacks rivals if and when they have an overwhelming advantage, and the recurrent pattern is that the pack with the most active fighters tends to win (Halfpenny, 2003).

Chimpanzees offer another illustration of the imbalance-of-power hypothesis and, as our close relatives, an especially important one. Chimpanzee societies engage in both "battles" and "raids" (Wilson, Hauser, & Wrangham, 2001; Wrangham & Wilson, 2003). In battles, roughly equal numbers of individuals of different groups face off against each other. Although there is a cacophony of screaming, thrashing, and charges, they tend to be non-lethal and involve few injuries. However, chimpanzees also engage in a rarer but very significant type of conflict: lethal raids (Goodall, 1986; Manson & Wrangham, 1991; Mitani, Watts, & Amsler, 2010; Wilson et al., 2014; Wrangham, 1999a). Here, attackers significantly outnumber opponents. Several members of a group will gather and move "uncharacteristically" quietly, "often with marked stealth", into a neighboring territory, which is never otherwise attempted (Manson & Wrangham, 1991, p. 370). If they find isolated individuals from the neighboring group, they will pursue and attack them. Unlike other primates, victims are often immobilized by one attacker, facilitating bites and blows by others. The raiding groups inflict multiple

[5] Jellicoe to Lanchester, 15/6/1916, held as B3/18, Lanchester archive, University of Coventry.

attacks in a Square Law scenario, and the victim is often killed or severely wounded.

Because battles are non-lethal and raids are lethal, we might predict a stronger selection pressure on Square Law encounters in chimpanzees. In support of this, in experiments simulating encounters with strangers of differing group sizes (using pre-recorded calls broadcast from unseen locations), chimpanzees' approach behavior appears to follow the predictions of Lanchester's Square Law, not the Linear Law (Wilson et al., 2002). Manson and Wrangham (1991) specifically argued that it was a low cost of coalitionary aggression, rather than exceptional benefits, which are critical to the selection of lethal raiding in chimpanzees and other species, an argument which is magnified by Lanchester's Square Law. Wrangham (1999a, p. 15) wrote that if 1 chimpanzee was killed out of a group of 10, then "its fighting power is reduced by 10%". His point was that this was already a big blow, with a long recovery time (male chimpanzees are philopatric so losses are only replaced by new births). But of course Lanchester's Square law makes such damage to fighting strength much more significant.

Moving to humans, we find a similar situation. Among small-scale indigenous societies, both battles and raids occur, and again they have very different characteristics and consequences (Chagnon, 1997; Gat, 2006; Keeley, 1996; Kelly, 2005; Meggitt, 1978; Wrangham, 1999b; Wrangham & Glowacki, 2012). Battles tend to be relatively rare, to involve approximately equal numbers on each side, to be highly ritualistic, and are not often lethal. Raids, by contrast, can be quite common, involve large asymmetries in the numbers on each side, and are often lethal. As a number of reviews of hunter-gatherer warfare have concurred (Gat, 2006; Otterbein, 2004; Turney-High, 1949; van der Dennen, 1995), "Everywhere the most frequent aim and expectation was to kill in a surprise attack" (Wrangham & Glowacki, 2012, p. 15). In a variety of ways, such raids facilitate Square Law combat.

Crucially, however, the phenomenon of raiding has significance over and above the mere *asymmetry* of numbers alone, because of the small *absolute* numbers involved. To explain why, it is first of all important to recall that in either of Lanchester's Laws, absolute numbers do not make any difference to who wins—only *relative* numbers, the force ratio, determine the outcome. However, in human evolutionary history it is likely that, among fights with the same force ratio – that is, between similar *relative* numbers, such as 3:1 or 30:10 or 300:100 – the outnumbered side would be better able to mitigate the larger side's Square Law numerical advantage when in fights of larger *absolute* size, by creating Linear Law conditions. Thus a given, biggish group of say 100 individuals may have a Square Law advantage against a smaller enemy, while against a larger enemy (from whom flight is impossible) the same group may be able to some extent to work together to prevent concentrated attacks, even in the absence of explicit command structures (*e.g.* by fighting shoulder-to-shoulder, back-to-back, or backs-to-the-wall, or by effective tactical maneuvers). This creates Linear Law conditions and negates its enemy's Square Law advantage. Thus a tightly-bonded group, committed to fighting with and for each other, is optimal.

Contrast this situation with an asymmetric raid, with *small* numbers involved. In such a raid – three individuals attacking one, say, in the extreme case – this is no longer possible: there is no one with whom to fight shoulder-to-shoulder, or back-to-back, nor contingents with whom to coordinate, cause diversions, or maneuver. In *small-scale* raids especially, ganging-up tells.

This distinction may help to explain why battles in small-scale societies are not particularly lethal. In a battle, any Square Law advantage for the larger side can (potentially) be mitigated as long as there are reasonable numbers of individuals on the smaller side, whereas in a raid the numbers or opportunity to respond effectively may be lacking. Plus, as we saw earlier, for an individual in the more numerous force death is a much less likely prospect in a Square Law raid than in a Linear Law battle, so it is unsurprising that the latter will tend to become a stand-off, especially in warfare below the military horizon, in which a force's coherence is more due to a calculation of individual prospects than to military discipline (Glowacki & Wrangham, 2013; Tooby & Cosmides, 1988, 2010). Indeed, an apparent equality of numbers may itself be a strong signal to avoid conflict. In purely mathematical terms, either of Lanchester's Laws would predict death to the last man. Fittingly, lethal raiding appears to be limited to mammals that have long-term social bonds (allowing coalitions) and variation in sub-group sizes within the broader social group (allowing asymmetries)—as occurs among wolves, lions, hyenas, chimpanzees, and humans (Manson & Wrangham, 1991; Wrangham, 1999a).

3.2. An example of a raid

To illustrate the important phenomenon of raiding, one of many possible examples is given by Samuel Hearne, an English explorer who joined a group of Chipewyan and Yellowknife Dene Indians on a journey from Hudson Bay to the Arctic Ocean in the summer of 1771. On reaching the Coppermine River in July, Dene scouts discovered an Inuit camp a few miles ahead. On this news, recounted Hearne, "their whole thoughts were immediately engaged in planning the best method of attack, and how they might steal on the poor Esquimaux the ensuing night, and kill them all while asleep" (Hearne, 1958, and following quotes). As well as having the advantage of surprise, the Dene judged themselves to have numerical superiority, as "[t]he number of my crew was so much greater than that which five tents could contain."

Hearne took a disparaging view of his fellow travelers' customs and organization, referring to them as an "undisciplined rabble … by no means accustomed to war or command." But he was clearly struck by the sudden coordination of action when conflict was at hand. The Indians acted quickly and "with the utmost uniformity of sentiment. There was not among them the least altercation or separate opinion; all were united in the general cause, and as ready to follow where Matonabbee [the group's leader] led … Never was reciprocity of interest more generally regarded among a number of people, than it was on the present occasion by my crew, for not one was a moment in want of anything that another could spare; and if ever the spirit of disinterested friendship expanded the heart of a Northern Indian, it was here exhibited in the most extensive meaning of the word."

They approached the camp with excruciating caution, taking a circuitous route along the lowest ground, wading through swamps, using the cover of rocks, until they were "within two hundred yards of the tents. There we lay in ambush for some time, watching the motions of the Esquimaux." They waited long into the night, finally descending on the village in a coordinated attack at around 1 am. Hearne, a distraught bystander to events, described how "the poor unhappy victims were surprised in the midst of their sleep, and had neither time nor power to make any resistance." All of them were killed, with no losses to the Dene.

A final point of note is the capture of an old Inuit man later the next day, who "fell a sacrifice to their fury: I verily believe not less than twenty had a hand in his death." Along with the element of surprise, any one of the Inuit clearly had little chance in the face of multiple attackers armed with guns and spears.

3.3. Lanchester's Square Law in early warfare

Here we describe how the imbalance-of-power hypothesis is given strong support by the Square Law, which we argue was the prevalent combat dynamic in early human lethal warfare.

Animal experiments have sometimes found the Linear Law to be a better fit in some species and settings than the Square Law (see Adams & Mesterton-Gibbons, 2003; Borges, 2002; Plowes & Adams, 2005). Lanchester's insight was not that the Square Law applies everywhere, but that either law may obtain depending on prevailing conditions. However, in nature, there does seem to be a taxonomic pattern. Wrangham's imbalance-of-power hypothesis (Manson & Wrangham, 1991; Wrangham, 1999a) suggests that among humans, as well as among chimpanzees and some social carnivores, lethal combat tends to occur when there is a strong asymmetry in the numbers on each side, and where multiple individuals can attack a few or solitary opponents at little risk to themselves. Groups come together when the opportunity arises to pick off subsets of the opposing side and exact a "defeat in detail" before the rest of the group can come to their aid.

The imbalance-of-power hypothesis can therefore be seen as an argument for the prevalence of Square Law conditions in deadly combat in our evolutionary history. But equally, the *implications* of Lanchester's Square Law not only support this hypothesis but extend it greatly, beyond the initial observation of the desirability of numerical superiority. First, recall that defeat in detail is especially stark under the Square Law, where it can reduce the number of casualties on the more numerous side to very low levels (in contrast to the Linear Law, under which it offers no special advantage). Secondly and crucially, Square Law conditions lower individual risks in participation and thus help to solve the especially severe collective action problem in warfare (Mathew & Boyd, 2011; Sosis et al., 2007; Tooby & Cosmides, 1988, 2010). Manson and Wrangham (1991, p. 371) explicitly argued that "unusually low" costs of aggression was a key condition for lethal raiding to evolve. Imbalances of power are much more significant than we might earlier have supposed—larger groups have an advantage, not merely in proportion to the imbalance, but disproportionately magnified by the Square Law.

Importantly, the Square Law may be *more* relevant to early human warfare than to the modern warfare for which it was originally conceived, because lethal conflict in our evolutionary past: (1) tended (most likely) to be conducted as surprise "raids" with major power asymmetries, as opposed to the staged "battles" of modern times anticipated by both sides; (2) tended to be "aimed" (in the Square Law sense, that all available lethal force is used against definite targets, rather than being dissipated randomly or left unemployed), in contrast to the density-dependent, Linear Law effects of many modern weapons; and (3) was conducted by fighters of similar fighting ability (α), compared with modern war in which differences in weapons mean fighting ability can vary by orders of magnitude.

This latter point suggests some further, indirect evidence of a role for Lanchester's Square Law in human evolution. If fighting was dominated by Square Law combat, then the fighting ability of individuals (α) would have been of secondary importance (numbers mattered more than muscle). This could partially account for the reduced sexual dimorphism in humans compared to other primates (Plavcan, 2001). There are also notable patterns across primates as a whole. In species that engage in one-on-one fights for mating opportunities, such as gorillas, orangutans, and mandrills, males are much larger than females. By contrast, among primate species in which coalitionary fighting is important, "selection for weaponry (canines) is reduced" (Plavcan, van Schaik, & Kappeler, 1995; Wilson et al., 2002, p. 1110). In particular, chimpanzees have small canines compared to other primates, which is exactly what we would expect if Square Law combat.

There are many subtleties in the application of the Square Law in an evolutionary context (see also Adams & Mesterton-Gibbons, 2003). First, one-on-one fights will also of course have been important in evolutionary history. Where these concerned reproductive access, they are likely to have had no small impact on Darwinian fitness. However, we suspect that coalitionary conflict is likely to have been as or more important. At, at least among chimpanzees and humans, within-group fighting over reproductive access is still often fought by and among coalitions, rather than individuals (de Waal, 1998; Harcourt & de Waal, 1992). Also, where they do occur, one-on-one fights are typically between members of the same group, and are not usually lethal. Although within-group killing is possible and does occur, it is tempered by a variety of social and ritual mechanisms (Boehm, 2001; Chagnon, 1997). Between groups, however, it is a different story, and even though wars may be rare, death rates from inter-group conflict in indigenous societies are thought to have been significant (Bowles, 2006; Keeley, 1996; Otterbein, 1989).

Secondly, it is important to note that, in fighting with bare hands or any weapons not instantly lethal, the "unit" in Lanchester's Laws need not be a life; rather it could be a single blow or injury, a large (but finite) number of which can be received before final incapacitation. Thus, for example, the implicit calculation of individual prospects, so much better for the outnumbering side under the Square Law, may be much more certain than would be the case in a gunfight, say. Unlike with modern, lethal weapons, the individual's chance of death in a raid may really be very low.

Thirdly, for the Square Law to apply it is crucial that simultaneous attacks – "ganging up" – be possible. In hand-to-hand combat without weapons there is clearly some limit to this, perhaps when the force ratio is three or four to one (only so many attackers can physically get at the victim). However, this is still amply sufficient. The crucial requirement for selection is variation, and in asymmetric fights in which the force ratio varies over an order of magnitude (from 3 or 4:1 to 1:3 or 4, say), the strong dependence of the casualty ratio on the force ratio under the Square Law is likely to create a significant selection pressure.

Finally, irrespective of whether or not the Square Law is reflected in human *psychology*, imbalances of power – and thus the Square Law – are likely to have shaped human *social organization*. In turn, aspects of social organization are also likely to have further augmented the Square Law. At the basic level, human groups can be much larger than those of most other social animals, providing many opportunities for interactions between groups of different sizes (Alexander, 1987), and thereby strong Square Law effects. The pressures of Square Law combat would have strongly favored a variety of traits, including social cohesion, the building of fortifications, the development of weapons, an ethic of loyalty in battle, and the formation of inter-group alliances (and in turn these would have enhanced the prominence of Square Law combat—via both its dangers and opportunities).[6] Other aspects of human evolution would also interact powerfully with the Square Law. Language allows groups of attackers to be readily summoned and coordinated, while intelligence allows foresight, planning, and tactics such as diversions and ambushes to assure numerical superiority. Indeed, it has been hypothesized that modern humans' capacity to apply social organization for military ends contributed to the evolution of large brains (the so-called "military intelligence hypothesis" Johnson, 2015) and was a crucial factor in the demise of the Neanderthals (Gat, 1999). Given that modern humans are always and everywhere good at creating group social cohesion (Hewstone, Rubin, & Willis, 2002; Kurzban & Neuberg, 2005), it would be unlikely if this trait failed to exert some advantage in intergroup competition and conflict (Bowles, 2009; Pitman, 2011; Tooby & Cosmides, 2010). Groups matter, and all the more so by virtue of Lanchester's Square Law.

3.4. The role of weapons

What happened with the advent of weapons? Early humans moved long ago from hand-held to projected weapons, ample time for these to have played a role in the evolution of human brains, behavior, and social organization. The Square Law does not require aiming of projectiles to be accurate. Rather, so that effects are proportional to the number of attackers (rather than to those attacked), it merely requires that the aimer have a definite target. To the extent this holds in lethal raiding, it reinforces Square Law conditions, enhancing the effects of group organization and coordination through simultaneous attack (Bingham, 2000; Bingham & Souza, 2009; Westergaard, Liv, Haynie, & Suomi, 2000).[7] Among the Mae Enga of New Guinea, for example, an "alert and agile"

[6] Roscoe (2008) raises the interesting idea that fortifications, at least in New Guinea, are not only meant to keep attackers out, but to pen them in if they manage to penetrate it. This way, they can be pursued and killed by the (many more) defenders inside. With an asymmetry of numbers in favor of the defenders, such fortifications exploit Lanchester's Square Law.

[7] For further discussion of this point, see the exchange of views following Boyd, Gintis, and Bowles (2010).

man can evade a single arrow fired from more than a few dozen yards away, but "the real danger in battle lies in the number of arrows simultaneously in the air", one of which is much more likely to strike the target (Meggitt, 1977, p. 56).

The use of tools for war appears to be uniquely human. Chimpanzees use tools and are powerful throwers, but these are not systematically exploited in conflict. Unlike other primates, by contrast, humans are accurate and lethal throwers (Westergaard et al., 2000), and hurling stones likely constituted the advent of projectile weaponry in conflict, perhaps several million years ago, long before purpose-made tools (Crosby, 2002).

More sophisticated projectile weapons are commonly used in indigenous cultures for hunting, and that was likely their origin. Their adaptation for fighting other humans is probably at least several hundred thousand years old. The oldest known weapons are the wooden "Schöningen Spears", which were found in Germany associated with animal remains, and dated to around 400,000 years old (Thieme, 1997). While the Schöningen Spears may have been used for stabbing rather than throwing, and the first stone projectile points are more recent, current evidence from skeletal functional morphology suggests an origin for thrown spears in the Middle Stone Age (Churchill & Rhodes, 2009). There may even be some evidence of cognitive adaptation to weapons. For example, humans have a bias to overestimate the speed of approaching objects (Neuhoff, 1998, 2001). We duck well before an object reaches us, which suggests an adaptive bias: the costs of ducking too soon are small, but the costs of ducking too late may be lethal.

3.5. The natural selection of a Square Law heuristic

Here we argue that the empirically observed assessment strategies that people employ are overwhelmingly consistent with the Square Law (Hypothesis 2 (H2)).

If the actual fighting strength of a given group is greater than it would appear to available visual stimuli (group size *squared* rather than group size *per se*), natural selection should have favored an assessment strategy that takes this phenomenon into account. Individuals who exploited Lanchester's Square Law would have taken on more fights that they were likely to win (with little risk to themselves), while avoiding letting fights between equal forces become lethal—increasing Darwinian fitness. Individuals who disregarded or discounted the Square Law would have missed out on opportunities to exploit others' weakness, conceded unnecessarily, entered losing battles, or incurred higher than expected costs—harming Darwinian fitness.

The simplest heuristic that could "correct" assessments to exploit the Square Law is to *overestimate* one's apparent advantage when on the larger side—"my gang of three is not three times stronger than my enemy, but nine times stronger," as measured by Lanchestrian fighting strength. Furthermore, the *degree* of bias should increase as groups get larger—confidence should scale disproportionately with group size. These two features are exactly what psychologists have found. A large literature in social psychology has discovered that individuals, especially men, exhibit "positive illusions" in assessments of their capabilities (Sharot, 2011; Taylor, 1989). Moreover, when they have these "illusions", they are more likely to fight (Johnson, 2004; Johnson, McDermott, Cowden, & Tingley, 2012; Johnson et al., 2006; Wrangham, 1999b). Most importantly, this effect appears to be especially pronounced *when in groups*. Numerous studies suggest that being in a group makes men particularly prone to confidence in their abilities, to engage in status competition, and to succumb to in-group/out-group thinking and feelings of superiority, narcissism, and aggression (Baumeister & Boden, 1998; Goleman, 1989; Janis, 1972; Postmes & Spears, 1998; Van Vugt, De Cremer, & Janssen, 2007; Wrangham & Wilson, 2004). All such phenomena are reduced when men act alone. In the opening quote, William McNeill described "a strange sense of personal enlargement; a sort of swelling out, becoming bigger than life, thanks to participation in a collective ritual" (McNeill, 1995, p. 2). Anyone who has been in a football stadium with 50,000 fans booming chants, cheers, and insults in unison knows the intoxicating power of this coalitionary confidence (Russell, 2008). This sense of exaggerated, disproportionate power when in a group is a common observation, and may be no illusion but rather, under the correct fighting conditions, an adaptive consequence of Lanchester's Square Law. As Rob Kurzban notes (Kurzban, 2012: 114), "as long as the judge is cold, hard reality", overconfidence is a bad idea. Interestingly, Lanchester's Square Law suggests that the cold, hard reality of the mathematics of conflict means that the remarkable confidence displayed in groups is not a mistake, but the way natural selection has maximized expected value (see also Johnson, Blumstein, Fowler, & Haselton, 2013). Such powerful emotions may be evolution's way of making sure our ambitions expand to take advantage of our true strength in numbers.

In a group, then, men may literally be more powerful than the sum of their parts, and they appear to act this way (even if they are not aware of it, or the reason why). Against a smaller enemy, Square Law logic suggests that a group of men *should* be more confident, and can afford to be more aggressive, because they *are* disproportionately powerful and more likely to win (Square Law) fights. Such a mindset may help to explain the extraordinary swagger, risk-taking, and tightly bonded coalitions among young men, gangs, military units, "honour" cultures, and sports crowds (Russell, 2008; Wrangham & Wilson, 2004).

4. Implications for modern war

While a Square Law heuristic may have been adaptive in the small-scale inter-group conflicts of our evolutionary past (for which we argue it was designed), it may be maladaptive in conflict today, and especially in modern warfare. An evolved Square Law heuristic may prompt aggression in modern settings where there is numerical asymmetry but where the underlying dynamics, owing to weapons or circumstances, are no longer Square Law. Of course an unarmed crowd is unlikely to attack a small number of troops with rifles, each individual having made an implicit calculation of the danger of leading such an attack. But the weapons may be the same on both sides and yet the underlying dynamics Linear-Law or worse. Modern conflicts and weapons typically do *not* conform to Lanchester's Square Law [e.g. MacKay (2011) for air combat; Johnson (1990) for infantry combat with automatic weapons]. When faced with non-Square Law battles, our cognitive machinery may blindly apply Square Law prejudices and goad us into thinking we are likely to win when in fact we are likely to lose, or at least to suffer inordinate costs in fighting.

A Square Law heuristic may therefore help to explain the prevalence of overconfidence on the eve of war. It is widely accepted that modern states often violate the expectations of rational choice theory in their decisions for war (Levy & Thompson, 2010; Sears, Huddy, & Jervis, 2003; Tetlock, 1998), and historians and political scientists have identified overconfidence (or "false optimism") as an especially important cause, recurring throughout history from the Peloponnesian War and World War I, to Vietnam and the 2003 Iraq War (Blainey, 1973; Levy, 1983; Van Evera, 1999). The belief that victory can be easy, quick and relatively painless has proven an especially dangerous attraction for statesmen past and present (Johnson, 2004; Walt, 2011). Quantitative analyses of wars among modern states show that, since 1500, initiators have lost one-quarter to one-half of the wars they started (Wang & Ray, 1994), and in the period since 1816, stronger states used to win most of the wars they initiated, but this has declined to the point that, in the last half-century, stronger states have tended to *lose* the wars they start (Arreguin-Toft, 2005). Other analyses argue that the benefits of modern wars, even for the victors, are outweighed by their costs (Van Evera, 1998; Waltz, 1979). Apparently, assessments of the costs and benefits of fighting are not well tuned to the modern environment, and an evolved Square Law heuristic offers one reason why.

A Square Law heuristic may have especially important implications for the kinds of wars being fought in the 21st century. In recent decades

Table 2
Conditions under which Lanchester's Square Law is more likely to obtain (than the Linear Law), and thus where an evolved Square Law heuristic would be advantageous.

Domain	Condition	Reason
Combat	Many fighting few	Square Law dynamics in operation
Weapons	Projectile	Multiple attacks easier, with less interference
Fire	Aimed	More likely to pick off targets
Asymmetry	Large	Other factors less likely to mitigate bigger mathematical picture
Environment	Open	Concentration of fire easier
Terrain[a]	Hilly	More angles onto target
Tactics	Ambush	Multiple synchronous attacks easier
Leadership[a]	Strong	Coordination and synchrony of attacks easier
Own deployment	Deployed so that all units can engage	Entire force brought to bear on each target
Enemy deployment	Divided into subgroups (for "defeat in detail")	Greater numerical superiority in each engagement

[a] These factors could also help to *mitigate* the effects of the Square Law when one is on the smaller side (and trying to avoid asymmetric combat, for example by dividing the enemy force or fighting in a confined area).

there has been a marked decline in conventional inter-state war, especially among the great powers (Mueller, 2004; Pinker, 2011). Counter-insurgency, interventions, and the so-called "new wars" mostly involve small-scale rogue states or non-state actors, such as terrorists, insurgents, warlords, or ethnic groups (Hoffman, 2009; Kaldor, 2002; Munkler, 2005; Strachan & Scheipers, 2011).[8] A defining feature of such warfare is a stark asymmetry of military power. One might think that the overwhelming force and sophisticated weapons of developed powers in such conflicts make for perfect Square Law superiority. But in fact this assumption may be false or even run in the opposite direction. The relevant variant of Lanchester's laws in the case of insurgency (Deitchman, 1962) is asymmetric single engagements, reflecting the insurgency's capacity to direct its fire while the state's efforts are more often unaimed, thwarted by the insurgency's advantages of dispersion and concealment. In many cases, the stronger force succeeds only in providing more targets (which, as discussed earlier, can steer things towards a Linear Law battle even with a much more numerous side). Further, at the *tactical* level, western soldiers are often numerically matched or outnumbered by insurgents who can pick and choose the time and place of battle, and mount an ambush, for example, only when they have superior numbers. In such situations, a Square Law heuristic among the more powerful side would be especially malicious. If an evolved Square Law heuristic was an advantage in human pre-history and some of recorded history, in modern war it may be as much a curse as a blessing.

This argues for a renewed emphasis on rational tactical thinking and operations research. If evolved intuitions about war are not to be relied upon, it becomes all the more important to analyze carefully the dynamics of a given military scenario. For example, in modern war a Square Law advantage is a matter not of mass but rather of concentration of fire (Fuller, 1926). Thus, at Rorke's Drift in 1879 a 150 strong contingent of the British Army was able to achieve Lanchestrian concentration of rifle fire against many thousands of Zulu opponents largely without such weapons, leading to a famous victory, in stark contrast to its crushing defeat when poorly deployed at Isandlwana the day before. Military training, discipline, cooperation, and leadership become especially important if one is to avoid intuitive pitfalls and unfavorable conditions, and to create favorable conditions instead. Sometimes, the Square Law will still obtain, or can be made to obtain, and where that is the case our evolved heuristics would help. Table 2 lays out some basic conditions in which combat is more likely to be Square Law. Above all, evolved assessment strategies may be maladaptive for large-scale lethal warfare, but remain adaptive when carrying out (strongly asymmetric) *raids*. This is consistent with an earlier quantitative analysis of data on modern war (mainly from World War II and the Arab-Israeli wars) which showed that raids were more likely to be won by the initiator than staged battles, even after force size discrepancies and other factors were statistically accounted for (Johnson, Wrangham, & Rosen, 2002). When we are involved in raids, we may find ourselves in our element.

5. Possible empirical tests

The possibility that humans have an evolved Square Law heuristic leads to a variety of predictions that future empirical and experimental studies could test (summarized in Table 3). The core prediction is that human assessments of fighting strength increase in proportion to group size *squared* (relative to opponent group size). An auxiliary prediction is that we should observe a Square Law heuristic among men but not (necessarily) among women, since they were less likely to have participated in inter-group conflict in our evolutionary past (Potts & Hayden, 2008; Wrangham & Peterson, 1996).[9]

5.1. Operationalization of variables

5.1.1. Dependent variables

Assessments (the dependent variable) could be operationalized in various ways: (1) assessments of fighting strength (e.g., "we are X times more powerful than them"); (2) expectations of victory ("we have an X chance of winning", in a stochastic Lanchester model); (3) numbers deployed for a given fight ("we need X people to win").

5.1.2. Independent variables

The independent variable is one's own group size, with the opposing group size as an additional independent (control) variable.[10] The functional dependence would then include a parameter (typically a power) whose estimate would enable discrimination between Hypothesis 1 (H1) and Hypothesis 2 (H2). Essentially, is the fitted curve linear or square?

5.1.3. Intervening variables

Intervening variables, which we predict to interact with the independent and dependent variables, include: (1) sex (as outlined above); and (2) fighting skill (of oneself and/or of one's group mates), indexed by body size or strength (since this parameter, α, affects both Linear and Square Law battles as well as numbers alone, and people

[8] Of course such wars may be more common today, but they are not new; there has been an insurgent dimension in most wars in history, including those which it is easy to view from the nation-state perspective. See Boot (2013), Kalyvas (2001) and Fleming (2009).

[9] Women have long been victims of war, of course, so they may have been subject to selective pressures acting on their kin-groups' decisions and performance in war. But it would be indirect. Other biological factors may mean the same trait is present in women as well, irrespective of its effective on female fitness, such as linkage disequilibrium. Nevertheless, the prediction stands as an interesting one to test.

[10] In fact, it remains an empirical question whether a Square Law heuristic would be anchored to raw (own group) numbers or relative numbers. While perfect information on both sides would be desirable, it may not be available (we can count our own number but often have to guess the numbers of the unseen enemy), or even necessary (in small-scale societies, enemy group sizes – e.g. raid victims, or a typical rival village – may have tended to be of roughly similar size, leaving one's own attacking contingent, alone, to be the most reliable indicator of relative numbers).

Table 3
Key methods, data, and variables to test for an evolved Square Law heuristic.

Method	Data	Dependent variables	Intervening variables	Independent variables
Lab experiment	Hypothetical scenarios	(1) Assessments of fighting strength	(1) Sex	(1) Group size (Hypothesis 1 (H1))
Empirical analysis	Wars	(2) Expectations about the ease of victory	(2) Fighting skill	(2) Group size squared (Hypothesis 2 (H2))
Proxy data	Battles, riots	(3) Numbers deployed to fight opponents of a given size		(3) Controls (e.g. opponent group size)

with greater fighting skill may perceive a greater utility in fighting over and above numerical disparities) (Sell, Tooby, & Cosmides, 2009).

5.2. Laboratory experiments

There are several ways these predictions could be tested experimentally. Most obviously, subjects could be asked to predict the relative strengths or outcomes of (several) virtual fights between groups of varying numerical size. The null hypothesis is that participants would display assessments consistent with the Linear Law and Hypothesis 1 (H1). More subtly, they could be asked to estimate their own (or one of the group's) chances of surviving the fight—here the null, Hypothesis 1 (H1) prediction is that this is the reciprocal of the force ratio ($1/k$).

Where there is any departure from the null hypothesis, however, the *shape* of the relationship would lead to interesting and divergent conclusions. Do assessments increase with the *square* of group size (especially among men), following the Square Law and Hypothesis 2 (H2)? Or are the data better fitted by an exponent between one and two, and significantly greater than one (some hybrid of Hypothesis 1 (H1) and Hypothesis 2 (H2))? Either outcome would support the argument that Lanchester's Square Law underlies, at least to some extent, the psychology of human conflict.

Such an experiment is not straightforward because of the vulnerability to Type II error. Identifying a specific slope is difficult if data are few or widely scattered, so a failure to find the predicted relationship might result because, while Hypothesis 2 (H2) is in fact true, the experimental paradigm did not fully elicit the effect, there is too much noise in the data, or because people's assessments are influenced by both evolved intuitive mechanisms (the Square Law) *and* learned cost-benefit calculations (which may follow the Linear Law, depending on their experience, the environment, or the experiment).

Finally, one could test for distinctions between fights with the same force *ratio* but differing absolute force *size* (e.g. 3:1, 30:10 or 300:100). This might support the dual advantages of a tightly bonded fighting group of reasonable size (Square Law when outnumbering its enemy, less so when itself outnumbered), which we discussed earlier. Indeed, with any departure from Hypothesis 1 (H1) we would have a novel evidence base from which to consider explanations of the importance of coalitions in human evolution (where the Square Law predominates in lethal combat), and for the origins of widespread military overconfidence observed in modern conflict (where the Square Law does not predominate).

5.3. Real world empirical studies

Experiments would be instructive, but even if they produced strong results, they would leave open the question of whether the same phenomenon occurs among real-world decision-makers in high stakes decisions on conflict—whether among hunter-gatherers, tribal societies, or modern states. However, this could be tested as well using empirical data. In all cases, the Hypothesis 2 (H2) prediction is that assessments of fighting strength and expectations about the ease of victory increase disproportionately with group size—that is, with group size squared (again controlling for relative numbers).

However, there are important challenges with the dependent variable in real-world tests. First, decisions for war are a one-off affair, so we only have one data point for each actor and war. This can be addressed by pooling data from multiple actors and wars, to reveal the overall relationship between assessments and force ratios, but there is likely to be considerable noise. Secondly, decision makers often do not give a clear assessment of their relative strength or chance of victory on the eve of conflict, or at all. Thirdly, we only know that actor X went to war with a given force ratio Y, so we do not know if they also would have gone to war with some other force ratio (in short, given other considerations and constraints, force ratios may have been of secondary importance or even unrelated to the decision to fight).

There are also important considerations for the independent variables. In empirical settings of large-scale conflict, there is a range of possible variables that may act as the *cue* for "group size". These may include, among others: (1) population; (2) military power (e.g. size of the army, or other military assets); (3) economic or industrial power; (4) the number or resources of allies; or (5) the size of the decision-making group. The latter, of course, should *not* affect one's decision to fight, because it is unrelated to success in war. But such cues might nevertheless influence our judgment and decision-making because, in our ancestral environment, one's own group size would have been indexed by the people immediately around us, not by the numbers of distantly deployed tanks or invisible armies.

5.4. Proxy data

Obtaining good data from the real world that are suitable to test the hypotheses (for example, from historical wars or hunter-gatherer warfare) is likely to be challenging. In order to test the hypothesis we would ideally have *multiple* estimates of fighting strength or the probability of winning for each subject, so we can see how their assessments and decisions *change* across a range of different relative force sizes—only this can reveal a square law heuristic in a given person. In launching a war, as noted, there is only one decision.

However, other kinds of proxy data may offer such repeated measures. For example, individuals lower down the chain do make multiple assessments if they are military commanders committing differently sized units to fight different numbers of opponents (they may make many such decisions over the course of a given campaign). The same is true of wargames and military exercises. In a non-military setting, different numbers of riot police are likely to be sent to deal with different sized crowds, and so on. Available sample sizes and the degree to which conditions can be controlled will be important factors in identifying appropriate data. One might also be able to look at explicit attrition rates (which would also reveal the laws at work). For example, in dodgeball, when there is a melee, people tend to go out faster (Mark Flinn, personal communication).

6. Conclusions

The predominant form of deadly conflict in human evolutionary history, as well as among chimpanzees and some social carnivores, appears to have been asymmetric raiding, in which an attacking group has an overwhelming numerical advantage and uses it to ambush and kill members of rival groups at little cost to themselves (Manson & Wrangham, 1991; Wrangham, 1999a). Attack is concentrated while defense is divided, leading to Lanchester's Square Law, under which the fighting strength of a group is proportional not to group size but to group size *squared*, with the risk to an individual attacker varying inversely with this.

If so, human assessment strategies have likely been subject to several million years of selection pressure under Lanchester's Square Law (as opposed to the Linear Law). Natural selection may thus have favored a heuristic that "corrects" assessments to take the consequences of Lanchester's Square Law into account. Social psychologists report that we observe exactly such a tendency: people (appear to) *overestimate* their capabilities, and this bias is accentuated among men, and especially pronounced when in groups (Baumeister & Boden, 1998; Goleman, 1989; Janis, 1972; Johnson et al., 2006; Wrangham & Wilson, 2004). Such a bias might seem detrimental but, at least in the context of fighting, by virtue of Lanchester's Laws of combat it may in fact serve to maximize expected utility (sensu Kurzban, 2012: 114)—and, in the past, Darwinian fitness.

Lanchester's Laws of Combat have several implications for human evolution. First, they may explain behavioral differences between our closest cousins, bonobos and chimpanzees. Though widely understood to be less aggressive *than chimpanzees*, aggression among bonobos is still common, within and between groups, and among both sexes (de Waal, 1989; de Waal & Lanting, 1997; Stanford, 1998; Wrangham, 1999a). Kanō (1992) reported that half of all bonobo encounters involved aggression. However, the contexts are different. Among bonobo males, aggression tends to be one-on-one interactions among members of the same group (a Linear Law scenario), whereas chimpanzees form coalitions in both within-and between group conflict, most notably ganging up in lethal asymmetric raids on neighboring groups (a Square Law scenario). This has profound implications for the two species' social ecology, since male coalitions become vital for chimpanzees but not for bonobos (Chapman, White, & Wrangham, 1994; McGrew, Marchant, & Nishida, 1996; Wrangham, 1986). The magnanimity of peace among bonobos may be partly due to the mathematics of war.

Secondly, Lanchester's Laws of combat suggest that coalitions and coalitionary psychology are even more important for Darwinian fitness than we previously thought. Whether or not we have an evolved Square Law *heuristic*, these fundamental laws of conflict underlie our *interactions*. Human social organization, if not the human brain itself, is likely to have been heavily shaped by the effects of Lanchester's Square Law. This would be worthy of further investigation. Coalitions, social bonds, alliances with other groups, technology, and strategy all gain a new significance.

Thirdly, however, our main argument has been that the human brain *has* been influenced by Lanchester's Laws of Combat: the predominance of Square Law combat in human evolutionary history is likely to have shaped our assessment mechanisms to take into account the advantage of ganging up (and the danger of being outnumbered). We may have evolved a "Square Law heuristic", such that when we assess our strengths and chances of success in inter-group conflict, we tend to see through square lenses.

Fourthly, there are implications for modern war. A Square Law heuristic may have been adaptive in the environment in which we evolved but, today, it is likely to make us overconfident. Modern weapons and forms of engagement often mean the Square Law does not apply. And even where it does apply, it may work in subtly paradoxical ways—for example, in favor of the ostensibly weaker side, such as insurgents.

Fifthly, the Square Law generates some interesting reverse predictions for those on the weaker side, implying a selection pressure for a very low estimate of one's chances when outnumbered. An exaggerated fear of a more numerous enemy, especially when alone or in a small group, may thus also be adaptive. When we are few and outnumbered, our enemies should loom very large indeed. Interestingly, Chagnon describes the Yanomamo as "terrified" of their neighbors, and LeBlanc reports that the Dani of highland New Guinea have recurrent dreams of being surrounded (personal communication). The "fear" of other states and "threat inflation" are also core concepts in international relations theory (e.g., Jervis, 1978; Thrall & Cramer, 2009).

Of course, although we have proposed that humans have an evolved Square Law heuristic, which may be maladaptive in some settings, this does not mean that we are incapable of adjusting our assessments to take account of other factors. Conscious assessment of available data will obviously also contribute to real-life decision-making (Kahneman, 2011; Keegan, 2002; van Creveld, 1985).[11] But the point is that evolved assessment strategies are likely to play a role as well.

We began this article with the defeat of Crazy Horse and his thousand warriors at the hands of 40 men in the Wagon Box Fight of 1867. By 1876, the tables would turn. The shock defeat of Colonel George A. Custer at the Battle of the Little Bighorn was due in no small part to a remarkable flanking maneuver by Lakota Chiefs Crazy Horse and Gall and their horsemen. This time the Sioux would engineer a situation in which their full complement of warriors would be free to attack US soldiers in the open. Time and again in the intervening years, battlefield direction of hundreds of independently minded Sioux braves had proved extremely difficult and the all-important military feats of surprise and flanking were rarely achieved. But Crazy Horse had learnt the lesson of the Wagon Box Fight: never to confront the disciplined and better-armed white soldiers head on. At the Little Bighorn he would bring this knowledge spectacularly into effect. Riding in a painstakingly long sweep up the valley, out of sight of Custer (who was himself attempting a flanking maneuver), Crazy Horse and Gall eventually hooked up into the hills where their thundering force broke the crest of a ridge to find themselves perfectly blocking their attacker's path. Standing before them was Custer and his contingent of 225 hardened soldiers of the 7th Cavalry—well armed, well trained, and experienced. Twenty minutes later every one of them lay dead.

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[11] Kahneman (2011) divides judgment and decision-making into "Type 1" thinking, which is intuitive, subconscious and fast, and "Type 2" thinking, which is effortful, conscious, and slow. Both contribute to decision-making outcomes. We suggest that evolved Square Law assessment strategies are an example of Type 1 thinking.

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